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not taken into the account in the article by Mr. Hill, neither is it, in the reasonings of Laplace on the time of the rotary motion of the earth on its axis, nor in the discussion of the subject of perturbations.

The bearing of fluidity upon the stability of the system and the great changes which are to take place, some probably in consequence thereof, are very clearly referred to in the book of Psalms, in the Prophets, by our Saviour himself in the Gospels of Matthew and Mark, by the Apostle Peter, and in the book of Revelation of the Apostle John.

ON THE SUMMATION OF SERIES.

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Suppose we have a series whose n^{th} term is of the form

$$\frac{u_m}{(a x + b)(a x + a + b)(a x + 2a + b) \dots (a x + n - 1 a + b)}$$

where u_m is an algebraic function of x of not greater than $n - 2$ dimensions: it can be summed very simply by an ordinary algebraic method in this manner:

Resolve the general term into partial fractions of the form

$$\frac{A}{a x + b} + \frac{B}{a x + a + b} + \frac{C}{a x + 2a + b} + \dots \&c.,$$

then we shall have $Au'_{n-1} + Bu''_{n-1} + Cu'''_{n-1} + \dots \&c = u_m$ where

$u'_{n-1}, u''_{n-1}, u'''_{n-1} \&c.$ are algebraic functions of x of the $n - 1^{\text{th}}$ degree.

If now these expressions be expanded, and we equate the coefficients of x^{n-1} we shall have in the case supposed $\therefore m$ is less than $n - 1$

$a^{n-1}(A + B + C + \&c.) = 0$ or $A + B + C + \&c = 0$.

$\therefore$ for $x = 1, 2, 3, 4 \&c.$ we shall have

$$\begin{aligned} &A \left\{ \frac{1}{a+b} + \frac{1}{2a+b} + \frac{1}{3a+b} + \frac{1}{4a+b} + \frac{1}{5a+b} + \&c., \right\} \\ &+ B \left\{ \frac{1}{2a+b} + \frac{1}{3a+b} + \frac{1}{4a+b} + \frac{1}{5a+b} + \&c., \right\} \\ &+ C \left\{ \frac{1}{3a+b} + \frac{1}{4a+b} + \frac{1}{5a+b} + \&c., \right\} \end{aligned}$$

whose summation will consist of a few only of the first and last terms.

As an example, let $n = 5, m = 3, a = 1 \& b = 8$ and take the series

$\frac{9}{9.10.11.12.13} + \frac{22}{10.11.12.13.14} + \frac{47}{11.12.13.14.15} + \frac{90}{12.13.14.15.16} +$
 &c., and let it be required to sum it to n terms and to infinity. We first find the scale of relation of the numerator to be $x^3 + 6x + 2$, which satisfies the condition of the problem. Let

$$\frac{A}{x+8} + \frac{B}{x+9} + \frac{C}{x+10} + \frac{D}{x+11} + \frac{E}{x+12} = \frac{x^3 + 6x + 2}{(x+8)(x+9)(x+10)(x+11)(x+12)}$$

$$A(x+9)(x+10)(x+11)(x+12) + B(x+8)(x+10)(x+11)(x+12) + \dots \&c. = x^3 + 6x + 2.$$

By making $x = -8, -9, -10, -11, -12$, respectively we have
 $A = -\frac{279}{12} \quad B = +\frac{1562}{12} \quad C = -\frac{3174}{12} \quad D = +\frac{2790}{12} \quad E = -\frac{899}{12}$
 and the series becomes, if S represents the sum,

$$12 S = -279 \left(\frac{1}{9} + \frac{1}{10} + \frac{1}{11} + \frac{1}{12} + \frac{1}{13} + \frac{1}{14} + \dots + \frac{1}{n+8} \right) \\
+ 1562 \left(\frac{1}{10} + \frac{1}{11} + \frac{1}{12} + \frac{1}{13} + \frac{1}{14} + \dots + \frac{1}{n+8} + \frac{1}{n+9} \right) \\
- 3174 \left(\frac{1}{11} + \frac{1}{12} + \frac{1}{13} + \frac{1}{14} + \dots + \frac{1}{n+8} + \frac{1}{n+9} + \frac{1}{n+10} \right) \\
+ 2790 \left(\frac{1}{12} + \frac{1}{13} + \frac{1}{14} + \dots + \frac{1}{n+8} + \frac{1}{n+9} + \frac{1}{n+10} + \frac{1}{n+11} \right) \\
- 899 \left(\frac{1}{13} + \frac{1}{14} + \dots + \frac{1}{n+8} + \frac{1}{n+9} + \frac{1}{n+10} + \frac{1}{n+11} + \frac{1}{n+12} \right)$$

$$12 S = \left(-\frac{279}{9} + \frac{279}{n+9} \right) + \left(\frac{1283}{10} - \frac{1283}{n+10} \right) \\
- \left(\frac{1891}{11} - \frac{1891}{n+11} \right) + \left(\frac{899}{12} - \frac{899}{n+12} \right)$$

to n terms,

$$12 \Sigma = -\frac{279}{9} + \frac{1283}{10} - \frac{1891}{11} + \frac{899}{12} = -31 + 128 - 172 + 83 \\
+ \frac{3}{10} + \frac{1}{11} + \frac{1}{4} = 8 \frac{141}{220};$$

for an infinite No. of terms

$$\therefore \Sigma = \frac{8 \frac{141}{220}}{12} = \frac{1901}{2640}.$$